An Investigation of Non-stationary Nature of Ice Accretion Data

Mahesh. D. Pandey[1] and N. Manzana[1]
[1]*Department of Civil Engineering, University of Waterloo, Waterloo, Ont., Canada N2L 3G1*
mdpandey@uwaterloo.ca, nmanzana@uwaterloo.ca

***Abstract*— The paper presents non-stationary extreme value analysis of ice accretion data obtained from Canadian meteorological stations. Non-stationary Gumbel distribution models are fitted to annual maximum values of the horizontal freezing precipitation and radial equivalent ice thickness data, calculated using Chaîné's model. Based on analysis of data from 3 airport stations, it is observed that horizontal precipitation exhibits small increasing trend, whereas the radial thickness data has no particular trend. Thus, in spite of presence of trend in historical time series of wind speed, temperature and precipitation data, the radial ice thickness data does appear to behave like a stationary process.**

Keywords—*Climate change, Ice accretion, Extreme value, Gumbel distribution, Non-stationary process, Structural safety*

I. Introduction

A. Background

There is an emerging concern about the impact of climate change effects on the safety of public infrastructure systems, such as buildings, bridges, electrical transmission lines, dams and dikes. If the occurrence rate and magnitude of extreme events, such as ice storms, high wind events, flash flooding and intense precipitation, are significantly affected by climate change, then the following two issues need to be addressed. Firstly, how to plan the reinforcement of existing infrastructure systems to maintain safety under the increased frequency and intensity of climate induced loads. Secondly, how to design structural systems at present to sustain increased loads resulting from future projected changes in the climate conditions. To create a knowledge base to address these two issues, researchers are relying on historical records of data as well as simulated loads under future climate scenarios projected by various climate models, such as [2] and [1].

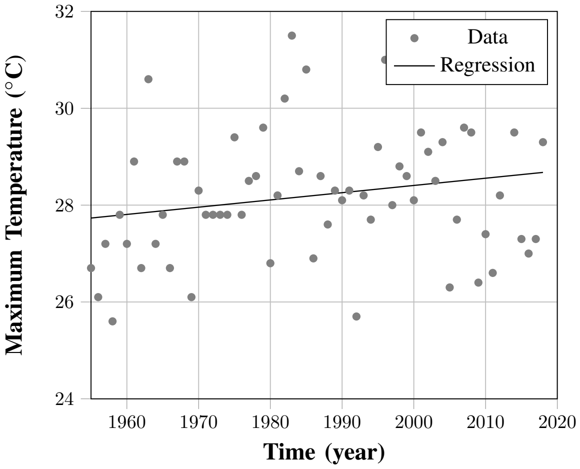
In water resources and environmental engineering, considerable efforts have been directed to examine the non-stationary nature of historical time series of data with the thinking that presence of non-stationary effects could provide some indication about the impact of climate change that has incurred in recent times, i.e., last 20-40 years. As an example, Fig. 1 shows increasing trends in annual extreme temperature data collected at St. John's airport. It is interesting that rate of increase of annual minimum temperature is larger than that of the maximum temperature. The maximum of gust wind speed data seem to suggest a decreasing trend with time, as shown in Fig. 2.

It is acknowledged that some trends in the data could also be indicative of changes in population distribution, land usage and new built-up infrastructure. Nevertheless, examination of historical data provides interesting insights in the nature of aggregate changes in climate data.

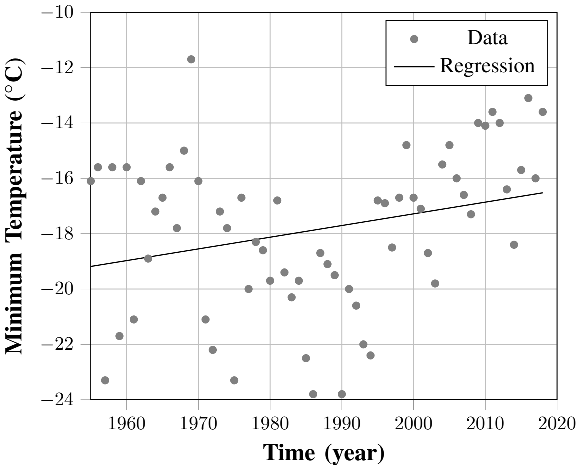
Since the safety and reliability of structures is largely affected by loads induced by wind, ice and rain, structural engineers have taken interest in investigating the presence of non-stationary effects in historical data of these variables. This provides motivation for this study to analyze ice accretion data collected at the Canadian meteorological stations.

B. Non-stationary Data Analysis: Literature

Several studies in the field of environmental and water resources engineering have developed non-stationary models of temperature, flood, and precipitation using historical data as well as simulated data obtained under



(a) Maximum temperature data



(b) Minimum temperature data

Fig. 1: Trends in annual extreme values of temperature data (St. John's Airport, NFL).

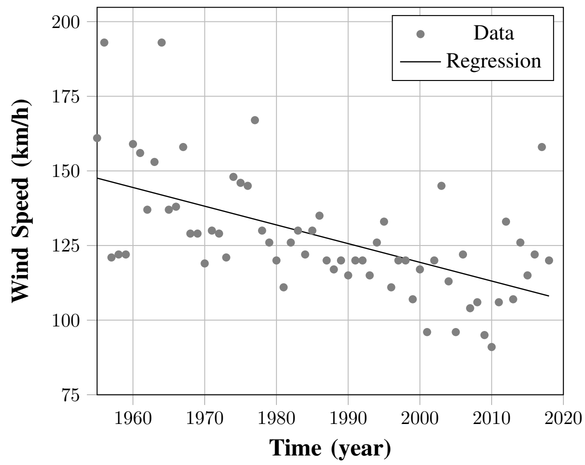


Fig. 2: The trend in annual maximum gust wind speed data (St. John's, NFL).

different scenarios of climate change. For example, non-stationary models of the river peak flow and precipitation data are reported in [9], [13]. It is not possible to review vast body of literature in this area. In this paper, the focus is on structural engineering literature on the effect of climate change on structural loads, which is rather limited at present.

The effect of climate change is likely to be seen in terms of increase in the frequency and intensity of natural hazards, resulting in increased infrastructure damage. Reference [11] presented risk-based life-cycle assessments of climate adaptation strategies for Australian housing under changing wind climate. This study utilized the climate projections made by the Australian Commonwealth Scientific and Industrial Research Organisation [1]. For example, CSIRO reported that for non-cyclonic winds, the mean wind speed may increase by up to 20% by the year 2070 along the east coast of Australia. The paper adopted the Gumbel distribution for extreme wind speed and assumed a linear trend for a given emission scenario.

A detailed analysis of wind and temperature data collected from Washington DC area is presented in [7]. This study showed a slight decrease in the annual maximum wind speeds in the last 50 to 70 years. The analysis of temperature data showed a marked increase in heat-wave days, and heat wave duration since 1980. In a recent study, a detailed literature review about effect of climate change on wind speed was presented in [8]. In this study, the Gumbel distribution was fitted to the wind speed data collected from three airports in the Washington DC area. The distribution parameters were then modified to account for a climate change scenario. For example, under the scenario RCP 8.5, the frequency of extreme wind environment is projected to increase by 0.06 days/year over a period of 100 years. No projections for change in the intensity of the wind climate were reported. This study reported a lack of conclusive evidence regarding current or future changes in extreme thunderstorm winds. However, there is a general view in the literature that increases in the frequency of thunderstorms are likely to occur in the future. This paper is a good source of references on the effect of climate change on the near surface wind speed distribution.

The effect of climate change in the Netherlands was summarized in terms of 4 scenarios of possible changes in temperature, precipitation, wind and sea level for a period of 30 years [10]. Based on these scenarios, the annual maximum daily mean wind speed is likely to change in 2050 (from 1990) within the range of -1% to +4%. In this study, the hourly mean wind speed in the Dutch climate is modelled as the Weibull distributed random variable with the shape and parameters of 1.6 and 5.7 m/s, respectively. This distribution corresponds to average speed of 5 m/s and a standard deviation of 3 m/s.

A non-stationary model for hurricane wind was presented in [6]. This study utilized hurricane data (1901-2010) collected from Miami-Dade County, Florida. This model was applied to compute the building damage cost due to hurricanes. The hurricane simulation model developed by [12] was modified in [5] to incorporate non-stationarity in the intensity and frequency of hurricanes.

The effect of climate change on design values of ice thickness under different scenarios of global mean temperature change is emerging as a topic of interest. [4] investigated potential changes in extreme ice loads in North America (NA) for future periods of specified global mean temperature change with reference to a recent 1986 - 2016 period. This study used simulated samples of the CanRCM4 regional climate model driven by CanESM2 under the RCP8.5 scenario. It was reported that 50 year return period value of ice thickness is likely to increase for most of northern NA and decreases for most of southern NA and some northeastern coastal regions. These changes are mainly caused by regional increases in future upper level and surface temperatures associated with global warming. Ice thickness distribution is also affected by changes in future precipitation intensity and surface wind speed.

Non-stationary modelling of historical ice accretion data has not been widely reported in the literature. Therefore, this study aims to analyze ice accretion data collected from Canadian meteorological stations with the purpose of investigating presence of any non-stationary effects in the data.

II. Data

The Environment and Climate Change Canada (ECCC) has developed a database of ice accretion data for over one hundred meteorological stations. The data include horizontal and vertical freezing precipitation as well as radial equivalent ice thickness on a conductor. The present study focuses on annual maximum values of the horizontal precipitation and the radial equivalent ice thickness calculated using the the Chaîné model [14].

In the Chaîné method, the increment in ice thickness on a conductor is calculated as follows:

$$\Delta R = -r + \left[r^2 + \frac{Kr}{2} \left(T_h^2 + T_v^2 \right)^{1/2} \right]^{1/2} \quad (1)$$

where ΔR is the incremental change in ice thickness on a radial surface, r is the radius of the cylinder (including ice already on conductor), K is a correction factor and T_h is the ice accretion rate on a horizontal surface. T_v is the ice accretion rate on a vertical surface, calculated as $T_v = 0.078VP^{0.88}$ where V is the wind speed and P=T_h is the freezing precipitation rate. Thus, this model depends on wind speed, precipitation and temperature, which tend to exhibit considerable variability. If the inputs to Chaîné's model exhibit non-stationary trends, then it is likely that ice accretion process could also some non-stationary trend. It is the purpose of this study to explore any such trend in the data.

This paper presents results of statistical analysis of ice accretion data for three stations, namely, St. John's International Airport (NFL), Toronto's Pearson International Airport (ONT) and Montreal's Pierre Elliott Trudeau International Airport (QUE). For these stations, time series of data are available from 1954 to 2018.

III. Non-stationary Gumbel Distribution Model

The origin of the Gumbel distribution lies in well-known asymptotic extreme value theory [3], which models the distribution of maximum value, M_n, of an *iid* sequence of random variables $X_1, X_2, \ldots, X_n$, as

$$\begin{aligned} \mathbb{P}(M_n \leq x) &= \mathbb{P}\left(X_1 \leq x, X_2 \leq x, \ldots, X_n \leq x\right) \\ &= \left[F_X(x)\right]^n, \quad (2) \end{aligned}$$

where $F_X(x)$ is the cumulative distribution function of X. As $n \to \infty$, there exist sequences of constant values a_n and b_n such that

$$\mathbb{P}\left(\frac{M_n - b_n}{a_n} \leq x\right) \to F_{\max}(x) \text{ as } n \to \infty \quad (3)$$

where $F_{\max}(x)$ is one of the three types of asymptotic extreme value distributions, often referred to as Gumbel, Fréchet or reverse Weibull distribution. The Gumbel distribution is widely used in modelling a sample of annual maximum values.

A general non-stationary version of the Gumbel distribution is defined as,

$$F_{\max}(x,t) = \exp\left(-\exp\left[-\frac{x-\alpha(t)}{\beta(t)}\right]\right), \alpha(t), \beta(t) > 0$$

where $\alpha(t)$ and $\beta(t)$ are time-dependent location and scale parameters, reflecting the presence of non-stationary effects in data. In this study, the scale parameter is assumed as a constant, and the location parameter is assumed to be a time-dependent function of the following forms:

$$\begin{aligned} \text{NS-G-Linear :} \quad & \alpha(t) = \alpha_0 + \alpha_1 t \\ \text{NS-G-Power-law :} \quad & \alpha(t) = \alpha_0 \cdot t^{\alpha_1}, \end{aligned} \quad (4)$$

The case of $\alpha(t) = \alpha$, denoted as S-G, represents usual stationary form of the Gumbel distribution. Note that the parameter, α_1, is intended to capture the degree of non-stationarity present in annual maximum ice thickness data. The maximum likelihood method is used to estimate all the parameters of the Gumbel model.

IV. Results

This section presents results of analysis of ice accretion data obtained from three airport stations, namely, St. John's (NFL), Toronto (ONT) and Montreal (QUE). Two sets of annual maximum data, namely, horizontal precipitation and radial equivalent ice thickness are fitted by the stationary Gumbel (S-G) and two non-stationary Gumbel models as discussed in the previous Section.

The median and 95th percentile estimated by stationary Gumbel models are presented in Table I.

ML estimates of the distribution parameters of horizontal precipitation and radial thickness are tabulated in Table II and Table III, respectively. Results are presented in Fig. 3-5. In each figure, data are plotted along with median curves obtained from the three Gumbel models. The median curve provides a visual indication of presence of any trend in the data.

Fig. 3 presents results for St. John's (NFL) ice thickness data. The horizontal precipitation shows very small increasing trend (α_1=0.014 for the linear NS-G model), whereas the radial thickness shows small decreasing trend.

Fig. 4 for Toronto station shows a small increasing trend in the horizontal precipitation (α_1=0.021 for the linear NS-G model), whereas the radial thickness data

TABLE I: Estimated Values of the Median and 95^{th} Percentile of Annual Ice Thickness Data (in mm): Stationary Gumbel Model

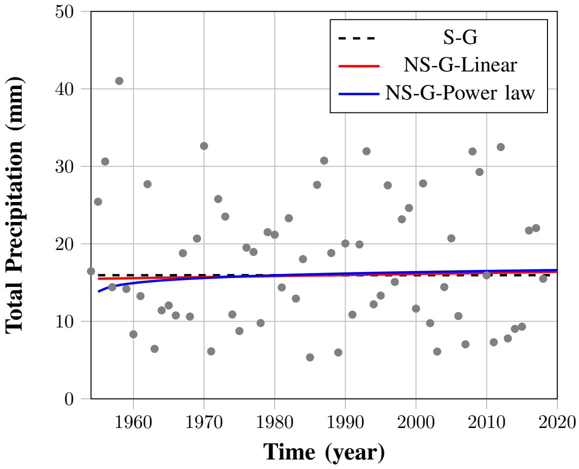
	Horizontal precipitation		Radial equivalent	
Station	median	95^{th} $\%ile$	median	95^{th} $\%ile$
St. John's	15.95	34.25	18.04	32.73
Toronto	5.87	15.94	6.87	17.71
Montreal	8.73	20.67	10.24	23.00

TABLE II: Parameter of the Gumbel Distribution of Horizontal Ice Precipitation

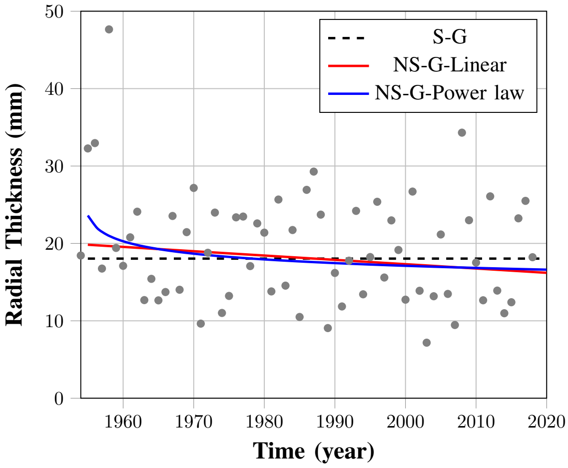
	Location function	α_0	α_1	β
St. John's	Constant	13.37		7.03
	Linear	12.93	0.014	7.02
	Power law	11.27	0.053	6.95
Toronto	Constant	4.46		3.87
	Linear	3.79	0.021	3.85
	Power law	3.03	0.12	3.82
Montreal	Constant	7.05		4.58
	Linear	6.63	0.013	4.58
	Power law	5.49	0.078	4.57

TABLE III: Parameter of the Gumbel Distribution of the Radial Equivalent Ice Thickness Distributions

	Location function	α_0	α_1	β
St. John's	Constant	15.97		5.65
	Linear	17.80	-0.056	5.54
	Power law	21.60	-0.094	5.46
Toronto	Constant	5.34		4.16
	Linear	5.56	-0.0067	4.16
	Power law	5.53	-0.011	4.16
Montreal	Constant	8.44		4.90
	Linear	8.68	-0.0073	4.89
	Power law	7.91	0.020	4.91

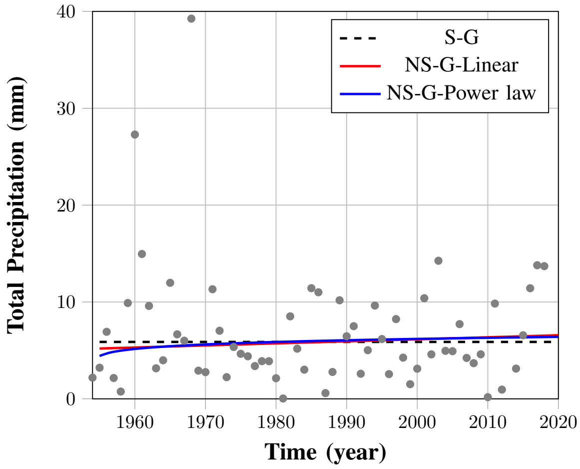


(a) Total horizontal precipitation

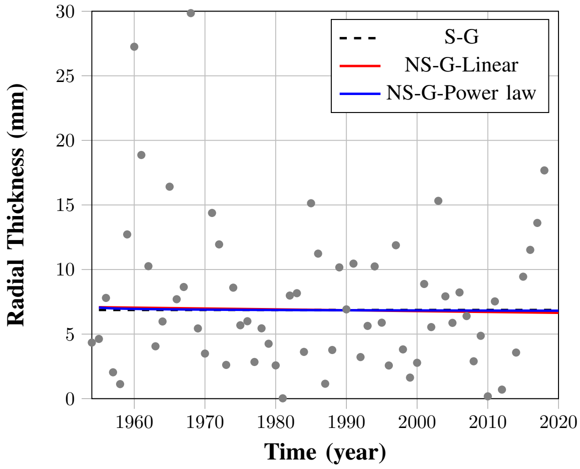


(b) Radial equivalent ice thickness

Fig. 3: Median curves of stationary and non-stationary Gumbel models fitted to annual maximum data (St. John's Airport).



(a) Total horizontal precipitation



(b) Radial equivalent ice thickness

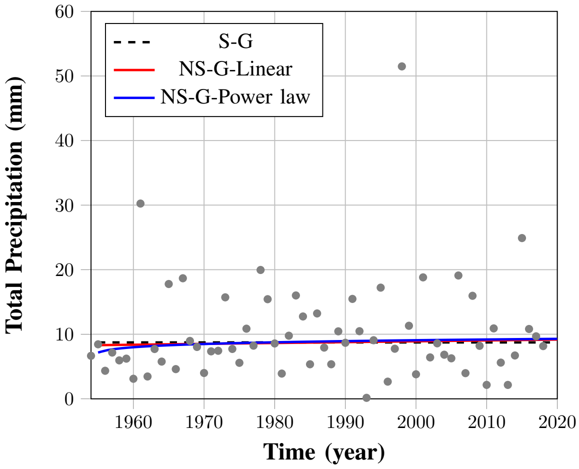
Fig. 4: Median curves of stationary and non-stationary Gumbel models (Toronto Pearson Airport).

exhibit no particular trend.

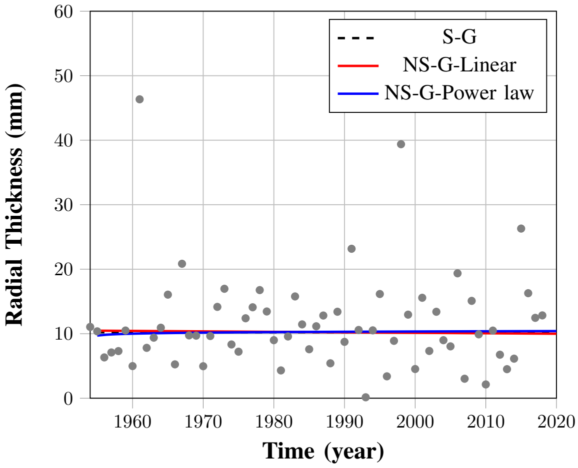
Fig. 5 for Montreal shows trends similar to that of Toronto data. There is a small increasing trend in the horizontal precipitation (α_1=0.013 for the linear NS-G model), whereas the radial thickness data exhibit no particular trend.

V. Conclusions

The present study explores the presence of any non-stationary trend in ice accretion data by analyzing annual maximum values of horizontal freezing precipitation and radial ice accretion data obtained from Canadian meteorological stations. Data are fitted by non-stationary Gumbel models with linear and power law variation of location parameter with time. Results are presented for 3 specific airport stations, namely, St. John's (NFL), Toronto (ONT) and Montreal (QUE). It is observed that horizontal precipitation exhibit very small increasing trend. The radial thickness data has no particular trend. Moreover, there is no discernable difference between the NS-G-Linear model and the NS-G-Power-Law model. It is interesting to note that in spite of presence of trend in historical time series of wind speed, temperature and precipitation data, the radial ice thickness data does behave like a stationary process.



(a) Total horizontal precipitation



(b) Radial equivalent ice thickness

Fig. 5: Stationary and non-stationary Gumbel models fitted to annual maximum data (Montreal Airport).

ACKNOWLEDGEMENT

The authors are grateful to Dr. R. Morris and Dr. P. Jarret, Scientists, Environment and Climate Change Canada, for providing ice accretion data collected from the Canadian meteorological stations. This data served as the basis for all statistical analysis results presented in the paper. The authors are thankful for the financial support of this study provided by the Natural Sciences Engineering Research Council of Canada (NSERC).

REFERENCES

[1] CSIRO. State of climate 2018. Technical Report ISBN: 978-1-925315-97, Bureau of Meteorology, Australia, 2018.

[2] ECCC. Canada's changing climate report. Technical report, Environment and Climate Change Canada, 2019.

[3] E. J. Gumbel. *Statistics of extremes*. Columbia University Press, New York, NY, 1959.

[4] D. Jeong, A. J. Cannon, and X. Zhang. Projected changes to extreme freezing precipitation and design ice loads over north america based on a large ensemble of canadian regional climate model simulations (under review). *Natural Hazards and Earth System Science*, 2018.

[5] J. Y. Lee and B. R. Ellingwood. A decision model for intergenerational life-cycle risk assessment of civil infrastructure exposed to hurricanes under climate change. *Reliability Engineering & System Safety*, 159:100–107, 2017.

[6] Q. W. Li, C. Wang, and H. Zhang. A probabilistic framework for hurricane damage assessment considering non-stationarity and correlation in hurricane actions. *Structural Safety*, 59:108–117, 2016.

[7] F. T. Lombardo and B. M. Ayyub. Analysis of washington, dc, wind and temperature extremes with examination of climate change for engineering applications. *ASCE-ASME J. Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, 1(1):04014005–1–11, 2015.

[8] F. T. Lombardo and B. M. Ayyub. Approach to estimating near-surface extreme wind speeds with climate change considerations. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, 3(3):1–11, 2017.

[9] J. Obeysekera and J. D. Salas. Frequency of recurrent extremes under nonstationarity. *Journal of Hydrologic Engineering*, 21(5):04016005, 2016.

[10] R. D. Steenbergen, T. Koster, and C. P. Geurts. The effect of climate change and natural variability on wind loading values for buildings. *Building and Environment*, 55:178–186, 2012.

[11] M. G. Stewart and X. Deng. Climate impact risks and climate adaptation engineering for built infrastructure. *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering*, 1(1):04014001(12), 2015.

[12] P. J. Vickery, P. F. Skerlj;, and L. A. Twisdale. Simulation of hurricane risk in the u.s. using empirical track model. *ASCE Journal of Structural Engineering*, 126(10):1222–1237, 2000.

[13] R. M. Vogel, C. Yaindl, and M. Walter. Nonstationarity: flood magnification and recurrence reduction factors in the United States. *Journal of the American Water Resources Association*, 47(3):464–474, 2011.

[14] T. C. Yip. Estimating icing amounts caused by freezing precipitation in canada. *Atmospheric Research*, 36:221–232, 1995.